Convex Optimization and Modeling	Jun.-Prof. Matthias Hein
Solution of Exercise Sheet 10	23.06.2010

Exercise 21 - Implementation of the barrier method

a. **(3 Points)** Implement the equality constrained Newton method as `NewtonEq.m`

b. **(2 Points)** Solve the following convex optimization problem. The objective is the same as on sheet 9.

$$f(x_1, x_2) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1+0.1}.$$

But now we have an equality constraint:

$$x_1 - x_2 = 1.$$

Plot the sequence of points (using the plotting routine provided in `Newton.m`).
Use as stopping criterion: $\lambda^2(x) < 10^{-8}$.
Use for stepsize selection: $\sigma = 0.2$, $\beta = 0.5$.

c. **(3 Points)** Implement the barrier method using the equality constrained Newton method as inner loop. Save as `Barrier.m`.

d. **(2 Points)** Solve the following convex optimization problem. The objective is the same as in exercise 10.

$$f(x_1, x_2) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1+0.1}.$$

But now we have box-constraints:

$$\|x - c\|_\infty \leq 1,$$

where $c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and an equality constraint:

$$x_1 - x_2 = 1.$$

Plot the central path (using the plotting routine provided in `Newton.m`). Same parameters as above.

Send the matlab-code and all plots (as `png`-files) to Shyam Rangapuram, email: `r.shyamsundar@gmail.com`.

Solution:

a. `the matlab-code is attached`

b. `The plot of the solution for a random feasible initial point:`

`Note, that for this problem the true solution is unknown. Therefore we approximate the optimal value` p^* `by the solution obtained for a very small` ε`. This is the reason why in the log plot of the difference of the current function value to` p^* `the plot levels off in the end.`

c. `matlab code can be found in the zip-file`

d. `The plot of the solution for a random feasible initial point:`

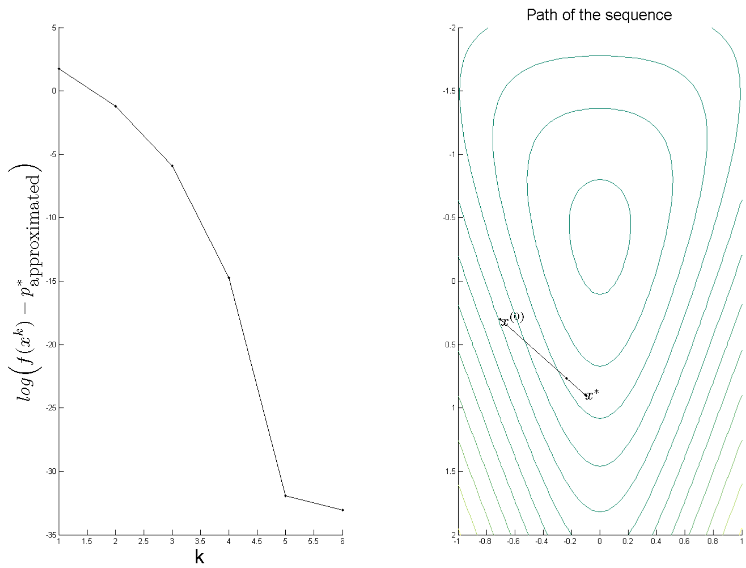

Figure 1: Solution path for the equality constrained problem with a random feasible initial starting point. The parameters are $\sigma = 0.2$, $\beta = 0.5$ and $\varepsilon = 10^{-8}$.

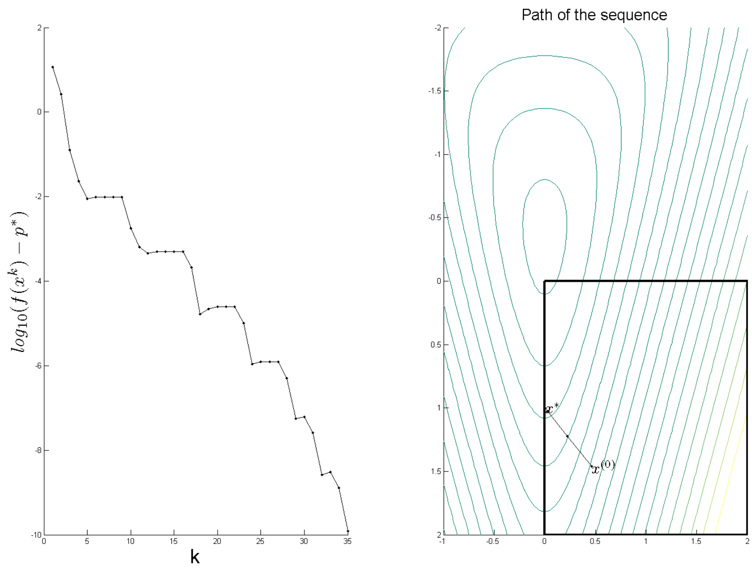

Figure 2: The central path (plus all Newton steps) are plotted for the parameters $\varepsilon_{\text{inner}} = 10^{-8}$, $\varepsilon_{\text{outer}} = 10^{-10}$ and $\alpha = 0.2$ and $\beta = 0.5$. Here the true optimum $x^* = (1, 0)$ is known, so that we can exactly obtain p^* for this problem. When you run the program you will observe that the KKT condition is not really fulfilled at the end. This seems to be due to numerical problems. The dual variable μ for the equality constraints is as it should be, but the dual variable λ constructed by $\lambda_i = \frac{1}{tg_i(x)}$ does not converge to the right value.