Convex Optimization and Modeling	Jun.-Prof. Matthias Hein
Exercise Sheet 3 - 28.4.2010	due: 5.5.2010

Exercise 6 - Convex, concave, quasiconvex and quasiconcave functions

a. **(3 Points)** Exercise 3.16a)-c) Determine for each function all the classes (convex, concave, quasiconvex, quasiconcave) to which it belongs.

b. **(2 Points)** Exercise 3.19a.

c. **(2 Bonus Points)** Exercise 3.13. (use the hint !). The Kullback-Leibler divergence $KL(p||q)$ is a classical measure of "distance" between two probability measures p and q. In the exercise it is used in a more general form for strictly positive measures on $\mathbb{R}^n_{++}$. Let $u, v \in \mathbb{R}^n_{++}$ (that means $u_i > 0$, $\forall\, i = 1, \ldots, n$)

$$KL(u||v) = \sum_{i=1}^{n} \Big[u_i \log\Big(\frac{u_i}{v_i}\Big) - u_i + v_i\Big].$$

The KL-divergence is a special case of the so called **Bregman-divergences** which have recently attracted some interest.

Exercise 7 - Subdifferential

- **(4 Points)** Derive the subdifferential of the general graph-based total variation energy functional,

$$F(f) = \sum_{i,j=1}^{n} w_{ij} |f_i - f_j|,$$

where $w \in \mathbb{R}^{n\times n}$ are the non-negative weights of a graph with n nodes and $f \in \mathbb{R}^n$ is a function on the nodes.

- **(2 Points)** Derive the subdifferential of $f : \mathbb{R}^n \to \mathbb{R}$ with $f(x) = \|x\|_\infty$.

Hints:

- for a) it might be helpful to first derive the subdifferential of the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = |x - y|$ using the chain rule introduced in the lecture.
- for b) make a case distinction if $x = 0$ or $x \neq 0$. For $x = 0$ the dual norm introduced in Exercise 1 is helpful and for $x \neq 0$ the direct use of the definition is required.