Convex Optimization and Modeling	Jun.-Prof. Matthias Hein
Exercise Sheet 11 - 23.06.2010	due: 30.06.2010

Exercise 22 - Projections onto convex sets

a. **(2 Points)** Derive the formula for the projection onto the L_1-ball shown in the lecture.

b. **(2 Points)** Derive the formula for the projection onto the positive semidefinite cone S_+^n shown in the lecture.

Hint:

- Use the KKT conditions for a).

Exercise 23 - Projected Gradient and Subgradient

Implement the projected subgradient and projected gradient method for the non-negative least squares (NNLS) problem.

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} \|Ax - b\|_2^2$$
$$x \succeq 0.$$

In both cases use $x^{(0)} = 0$ as starting vector.

a. **(3 Points)** `[xmin,fmin]=ProjectedSubgradientNNLS(MAXITER,A,b)`, where `MAXITER` is the maximal number of steps (there is no stopping criterion - so this is equal to the number of steps). Use a diminishing stepsize $\alpha^k = \frac{c}{k}$. What is a good choice of c ?

b. **(3 Points)** `[xmin,fmin]=ProjectedGradientNNLS(MAXITER,A,b)`. Use the constant stepsize version with the optimal estimate of the Lipschitz constant. Use $\left\| P_C\left(x^{(k)} - \frac{1}{L}\nabla f(x^{(k)})\right) - x^{(k)} \right\| \leq 10^{-10}$ as stopping criterion (together with the upper bound on the number of steps).

c. **(2 Points)** Run both methods with `MAXITER=100000` for the data from Exercise Sheet 9. Plot the logarithm of the relative error, $\log_{10}(\frac{f(x^k)}{p^*} - 1)$, where $p^* = 7.246560956533597 * 10^5$, for both methods into one plot. Why is the initial solution sparse ? Make the solution x^* even sparser by setting all components of x^* to zero which are smaller than 10. Plot the fit $X * x^*$ versus the mass-spectrogram `Y` in one figure (`hold on` command). What is the effect of the thresholding ?

d. **(2 Points)** The nice property of the projected subgradient method is that one can directly apply it to non-smooth objectives. How would the iterate look like for minimizing

$$\|Ax - b\|_2 + \lambda \|Dx\|_1 ,$$

under the non-negativity constraint (total variation denoising/deblurring with non-negativity constraint) ?

Send the matlab-code and all plots (as `png`-files) to Shyam Rangapuram, email: `r.shyamsundar@gmail.com`.