Convex Optimization and Modeling	Jun.-Prof. Matthias Hein
Exercise Sheet 7 - 26.05.2010	due: 02.06.2010

Exercise 17 - Relaxations of integer programming problems

There are two ways to get a lower bound for the optimal value of a combinatorial optimization problem. First, the dual problem is always convex and provides a lower bound for p^* by weak duality. Second, one relaxes the constraints e.g. instead of $x \in \{0,1\}$ one allows $x \in [0,1]$ and derives a continuous optimization problem (which might even be convex).

a. **(4 Points)** Exercise 5.13

b. **(4 Points)** Exercise 5.39

Hints:

- 5.13.a) note that the resulting Lagrangian is non-convex and thus $\nabla_x L$ is not sufficient for a global optimum.
- 5.13.b) optimize over the dual variables for the equality constraints in order to see equivalence of both dual problems.
- 5.39: Any rank-one matrix X can be written as $X = uv^T$ for some vectors u, v.

Exercise 18 - Differentiable approximation of l_1-norm minimization

This exercise discusses a common technique where one replaces a non-smooth objective function with a smoothed version. The critical question is how good the solution of the smoothed version is with respect to the original problem.

a. **(4 Points)** Exercise 6.4

Hints:

- For 6.4a) you can use the following steps
  - Derive the (necessary and sufficient) condition for a minimum of

    $$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{m} \phi\Big(\langle a_i, x \rangle - b_i \Big),$$

    where $\phi(u) = \sqrt{u^2 + \varepsilon}$.
  - Derive the dual problem of

    $$\min_{x \in \mathbb{R}^n,\, y \in \mathbb{R}^m} \|y\|_1$$
    $$\text{subject to: } Ax - b = y,$$

    where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. You need Hölders inequality with $p = 1$ and $q = \infty$.
  - Derive from the first step a dual feasible point and use that to derive a lower bound on p^*.