Convex Optimization and Modeling

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Exercise Sheet 12 - 30.06.2010

Exercise 24 - Proximity Operator

Prove the two statements in the lecture about the proximity operator prox_f defined as

$$\operatorname{prox}_{f} x = \underset{y \in \mathbb{R}^{n}}{\operatorname{arg\,min}} f(y) + \frac{1}{2} \|x - y\|^{2}.$$

- a. (2 Points) $x^* = \text{prox}_f x^*$ if and only if x^* is a minimizer of f. This means the set of fixed points of prox_f is equal to the set of minimizers of f.
- b. (3 Points) The proximity operator is non-expansive,

$$\left\|\operatorname{prox}_{f} x - \operatorname{prox}_{f} y\right\| \leq \left\|x - y\right\|.$$

Hint:

• In both cases use the optimality condition of the minimization problem. For the second problem use the same steps as for the projection onto convex sets.

Exercise 25 - Total Variation for general graphs

The goal of this exercise is to implement the total variation denoising scheme for general graphs developed in the lecture.

- a. (3 Points) Prove that $||A||^2 \le 4 \max_r \sum_{j=1}^n w_{rj}^2$.
- b. (1 Point) Show that $(A^T f)_{ij} = w_{ij}(f_i f_j)$.
- c. (7 Points) Implement the total variation method as introduced in the lecture as

f=TotalVariation(MAXITER,epsT,Y,W,lambda)

where $Y, f \in \mathbb{R}^{nm}$ are the input and output images represented as a vectors (use reshape), $W \in \mathbb{R}^{mn \times mn}$ encodes the weights between the pixels, and lambda is the regularization parameter. Use the box constraints $C = [0, 1]^n$.

- use $(f(x^k) f(x^{k+1}))/f(x^k) < 10^{-5} = epsT$ as stopping criterion or number of steps larger than MAXITER = 1000,
- Analyze the dependence of the number of required iterations on the employed regularization parameter λ .
- Which λ yields the best results in terms of reconstruction error and which in terms of visual appearance ?

Hints:

• make sure that all the matrices you use have sparse format (generate them with the command sparse). You can obtain the row and column indices, and the values of the non-zero entries of a sparse matrix via: [ix,jx,val]=sparse(W).

Send the matlab-code and all plots (as png-files) to Shyam Rangapuram, email: r.shyamsundar@gmail.com.