A Flexible Tensor Block Coordinate Ascent Scheme for Hypergraph Matching

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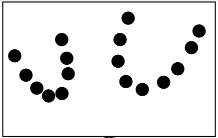
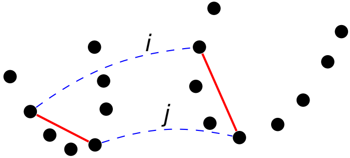
IEEE CVPR 2015, Boston, USA

Find correspondences between two sets of features / points?



Challenges: noise, deformation, geometric transformations

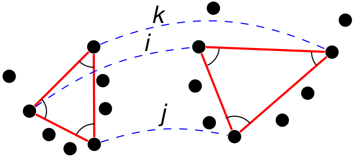
Graph Matching



- difficult to deal with rotation + scale
- Quadratic Assignment Problem

$$\max_{x \in M} \sum_{ij} \mathcal{F}_{ij} x_i x_j$$

Hypergraph Matching



- both rotation and scale invariant
- more robust to noise
- Higher Order Assignment Problem

$$\max_{x \in M} \sum_{ijk} \mathcal{F}^3_{ijk} x_i x_j x_k$$

M = discrete one-to-one mapping constraint

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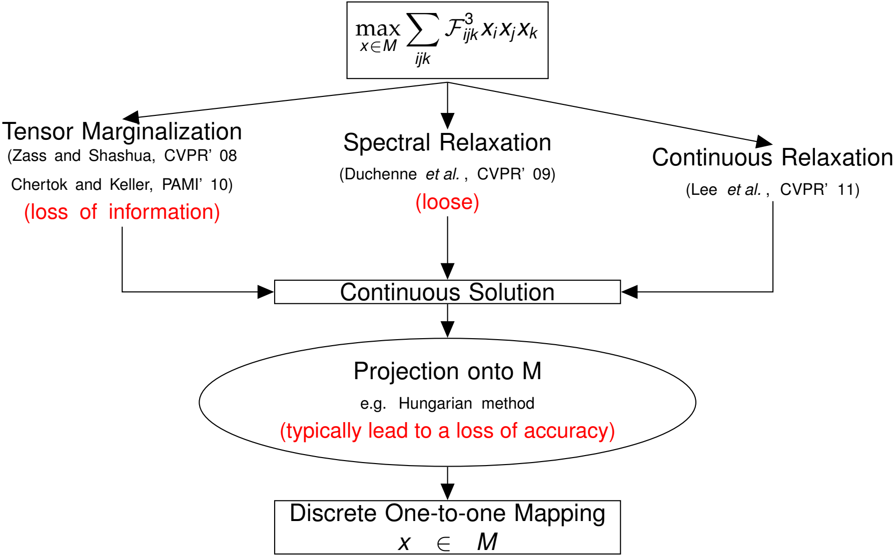


Main Concepts

- An m-dimensional array $\mathcal{F}^m$ is called an m-order tensor

  e.g. a vector is an one-dimensional tensor, a matrix is a two-dimensional tensor, etc

- One associate to any tensor $\mathcal{F}^m$ a multilinear form $F^m : \mathbb{R}^n \times \ldots \times \mathbb{R}^n \to \mathbb{R}$ such that

$$F^m(x^1, \ldots, x^m) := \sum_{i_1 \ldots i_m} \mathcal{F}^m_{i_1 \ldots i_m} x^1_{i_1} \ldots x^m_{i_m} \tag{1}$$

  When $x^1 = \ldots = x^m = x$, the resulting function

$$F^m(x, \ldots, x) := \sum_{i_1 \ldots i_m} \mathcal{F}^m_{i_1 \ldots i_m} x_{i_1} \ldots x_{i_m} \tag{2}$$

  is called the m-order score function

Hypergraph Matching problem becomes:

$$\max_{x \in M} \sum_{ijk} \mathcal{F}^3_{ijk} x_i x_j x_k = \boxed{\max_{x \in M} F^3(x, x, x)} \tag{3}$$

Step 1: Lifting the tensor

Given $\mathcal{F}^3 \in \mathbb{R}^{n\times n\times n}$, the lifted 4th order tensor $\mathcal{F}^4$ is defined as

$$\mathcal{F}^4_{ijkl} = \mathcal{F}^3_{ijk} + \mathcal{F}^3_{ijl} + \mathcal{F}^3_{ikl} + \mathcal{F}^3_{jkl} \quad (1)$$

$$\Longrightarrow F^4(x,x,x,x) = F^3(x,x,x)\sum_{i=1}^{n} x_i \quad (2)$$

Properties: If the new score function $F^4(x,x,x,x)$ is convex on $\mathbb{R}^n$ then

$$\boxed{\max_{x,y,z,t\in M} F^4(x,y,z,t) = \max_{x,y\in M} F^4(x,x,y,y) = \max_{x\in M} F^4(x,x,x,x)} \quad (3)$$

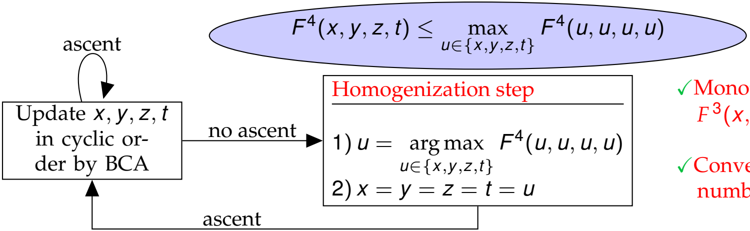
Remarks:

1. Since $\sum_i x_i = \text{constant} \quad \forall x \in M$, it holds from (2)

$$\max_{x\in M} F^4(x,x,x,x) \equiv \max_{x\in M} F^3(x,x,x)$$

2. The lifting step does not cause computational overload

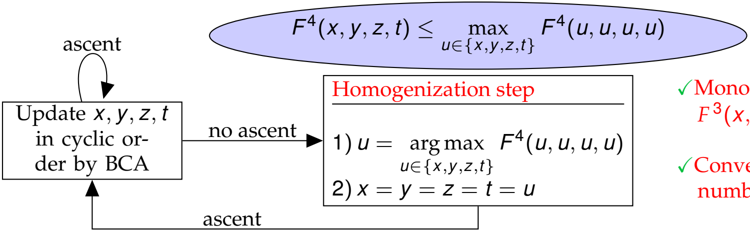
Algorithm 1: $\max_{x,y,z,t \in M} F^4(x, y, z, t)$



✓Monotonic ascent for $F^3(x,x,x)$ on M

✓Convergence in finite number of iterations

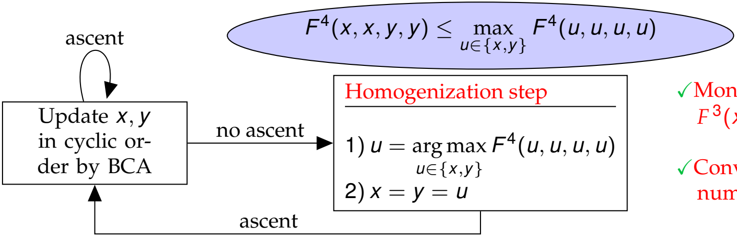
Algorithm 1: $\max_{x,y,z,t \in M} F^4(x, y, z, t)$



✓Monotonic ascent for $F^3(x, x, x)$ on M

✓Convergence in finite number of iterations

Algorithm 2: $\max_{x,y \in M} F^4(x, x, y, y)$



✓Monotonic ascent for $F^3(x, x, x)$ on M

✓Convergence in finite number of iterations

Step 2: Tackling the convexity assumption

Consider the following multilinear form:

$$F^4_\alpha(x,y,z,t) := F^4(x,y,z,t) + \alpha \frac{\langle x,y\rangle\langle z,t\rangle + \langle x,z\rangle\langle y,t\rangle + \langle x,t\rangle\langle y,z\rangle}{3} \quad (1)$$

then $F^4_\alpha(x,x,x,x)$ is convex for any $\alpha \geq 3\sqrt{\sum_{i,j,k,l=1}^{n} (\mathcal{F}^4_{ijkl})^2}$. Besides, it holds for all α

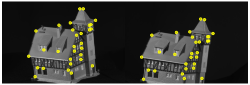
$$\max_{x\in M} F^4_\alpha(x,x,x,x) \equiv \max_{x\in M} F^3(x,x,x) \quad (2)$$

To reduce the influence of α:

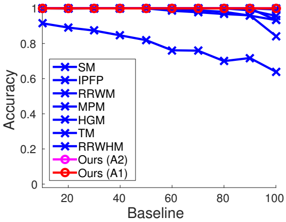
- Phase 1: run algorithm with $\alpha = 0$
- Phase 2: restart algorithm with $\alpha = 3\sqrt{\sum_{i,j,k,l=1}^{n} (\mathcal{F}^4_{ijkl})^2}$ with starting point initialized from output of Phase 1

Matching Quality (CMU House Dataset)

- 30 points are manually tracked over a sequence of frames of the same object taken at different view points
- In Figures: $\mathrm{Baseline} =$ difficulty of the problem



30 points vs. 30 points



(Easy)

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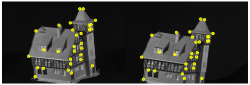
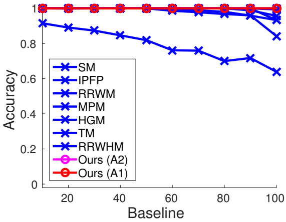
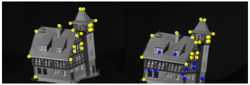


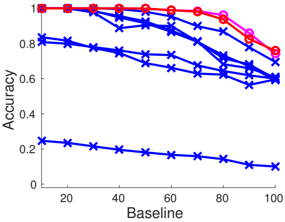
30 points vs. 30 points



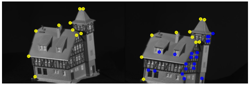
(Easy)



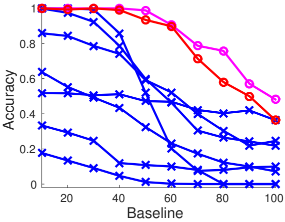
20 points vs. 30 points



(Medium)

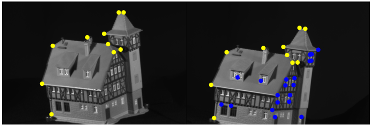


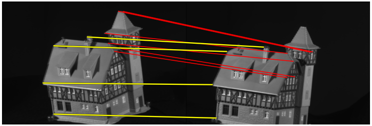
10 points vs. 30 points

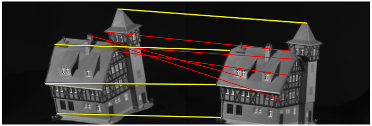


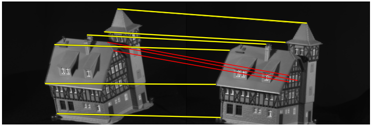
(Difficult)

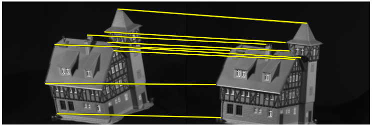
Matching Quality (CMU House Dataset)

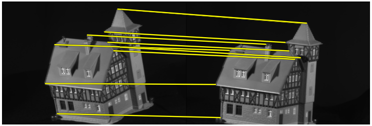

10 points vs 30 points


MPM (4/10)

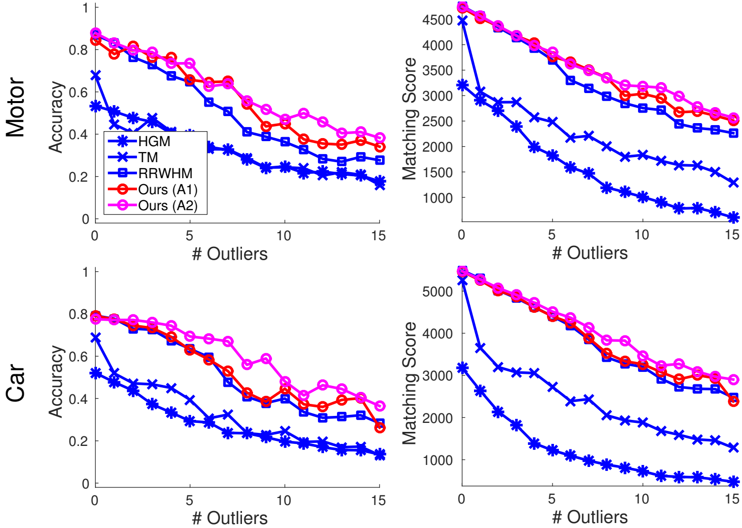

TM (5/10)


RRWHM (7/10)

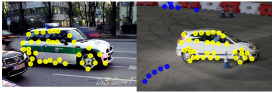

ours(A1) (10/10)


ours(A2) (10/10)

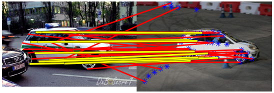
Matching Quality (Car and Motorbike Dataset)



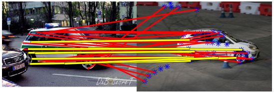
Matching Quality (Car and Motorbike Dataset)



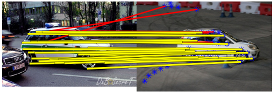
Example Input



TM (10/34)

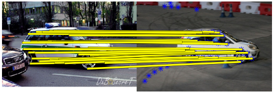


HGM (9/34)



RRWHM (28/34)

Ours(A1) (28/34)



Ours(A2) (34/34)

Conclusion

Our Contributions

- a theoretically sound optimization framework for hypergraph matching
- two state-of-the-art algorithms with theoretical guarantees
  - monotonic ascent for original score function on M
  - convergence in finite number of iterations

Main Ideas

- lifting 3rd order tensor to 4th order higher order
- multilinear optimization over one-to-one constraint

$$\max_{\mathbf{x}\in M} F^3(\mathbf{x},\mathbf{x},\mathbf{x}) \rightarrow \max_{\mathbf{x}\in M} F^4(\mathbf{x},\mathbf{x},\mathbf{x},\mathbf{x}) \rightarrow \max_{\mathbf{x}\in M} F^4_\alpha(\mathbf{x},\mathbf{x},\mathbf{x},\mathbf{x})$$

$$\rightarrow \max_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{t}\in M} F^4_\alpha(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{t}) \rightarrow \text{Algorithm 1}$$

$$\rightarrow \max_{\mathbf{x},\mathbf{y}\in M} F^4_\alpha(\mathbf{x},\mathbf{x},\mathbf{y},\mathbf{y}) \rightarrow \text{Algorithm 2}$$

Advantages

- no projection at final step
- flexible objective function
- simple yet effective